CE 261: Fluid Mechanics

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CHAPTER 3

KINEMATICS OF FLUID FLOW

TYPES OF FLUID

- Ideal fluid
- Real fluid
- Compressible fluid
- Incompressible fluid
- Newtonian fluid
- Non-Newtonian fluid

TYPES OF FLUID

Ideal Fluid: has no Viscosity

Real Fluid: Whenever motion takes places, shearing forces are developed

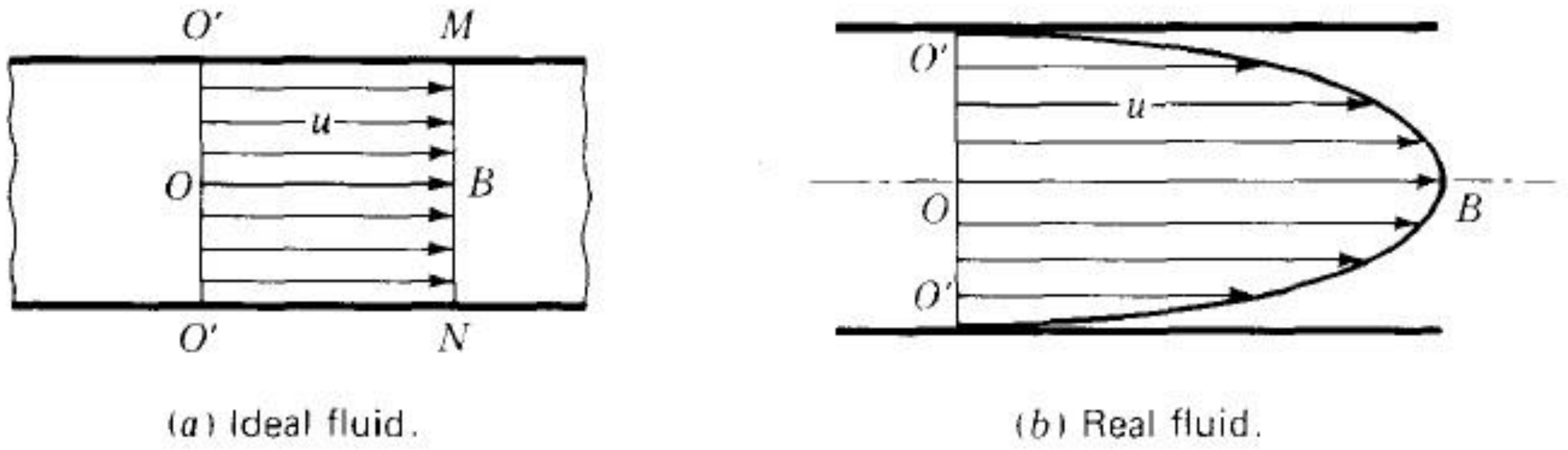


Figure 3.1. Typical velocity profiles. (*a*) Ideal fluid. (*b*) Real fluid.

TYPES OF FLUID

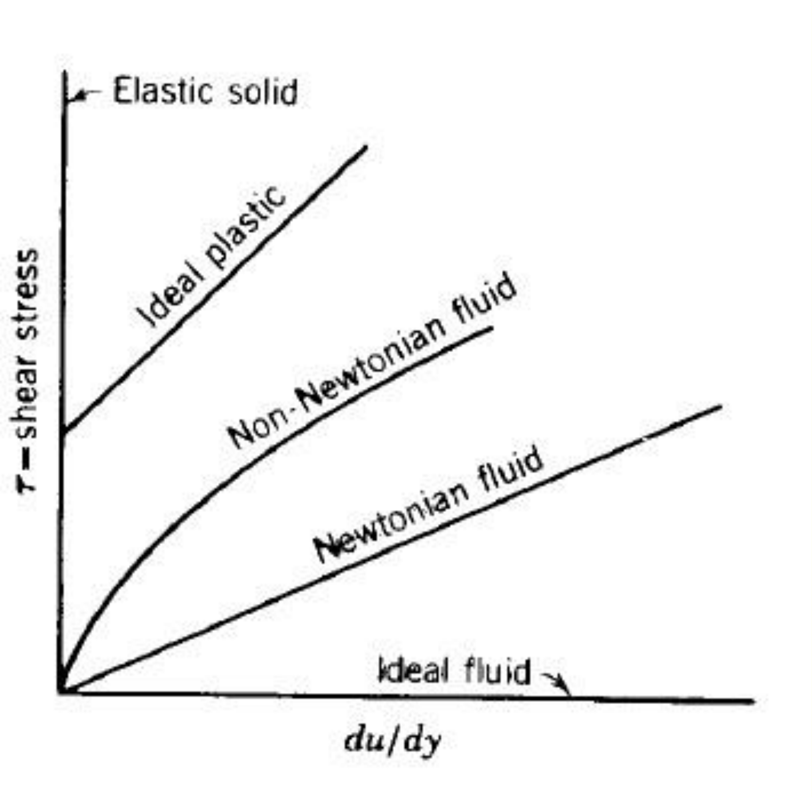
Incompressible fluid: Fluid with constant density with change in pressure

Compressible fluid: Fluid with variable density

TYPES OF FLUID

Newtonian fluid: A fluid for which viscosity does not change with rate of deformation

Non-Newtonian fluid: Under force it becomes more liquid or more solid.



TYPES OF FLOW

- Laminar flow and Turbulent flow
- Steady and Unsteady Flow
- Uniform and Non-Uniform Flow
- One, Two and Three Dimensional Flow

REYNOLD’S NUMBER

Reynold’s experiment: https://youtu.be/pae5WrmDzUU

$$Re = \frac{Inertia\ Force}{Viscous\ Force}$$

$$Re = \frac{\rho V L}{\mu}$$

TYPES OF FLOW

Laminar flow: Type of fluid flow in which the fluid travels smoothly or in regular paths.

Turbulent flow: fluid undergoes irregular fluctuations and mixing

TYPES OF FLOW

Steady Flow: Flow properties remain constant with respect to time

Unsteady Flow: Flow properties vary with respect to time.

TYPES OF FLOW

Uniform Flow: If the flow velocity is assumed to have the same speed and direction at every point within the fluid, it is said to be uniform.

Non-Uniform Flow: If at a given instant, the velocity is not the same at every point, the flow is non–uniform.

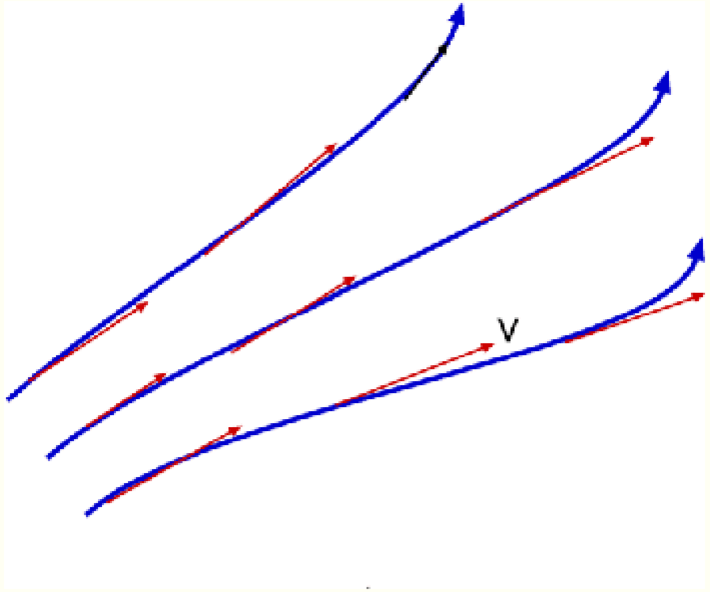
FLOW PATTERN

- Streamline
- Streamtube
- Pathline
- Streakline

FLOW PATTERN

Streamline

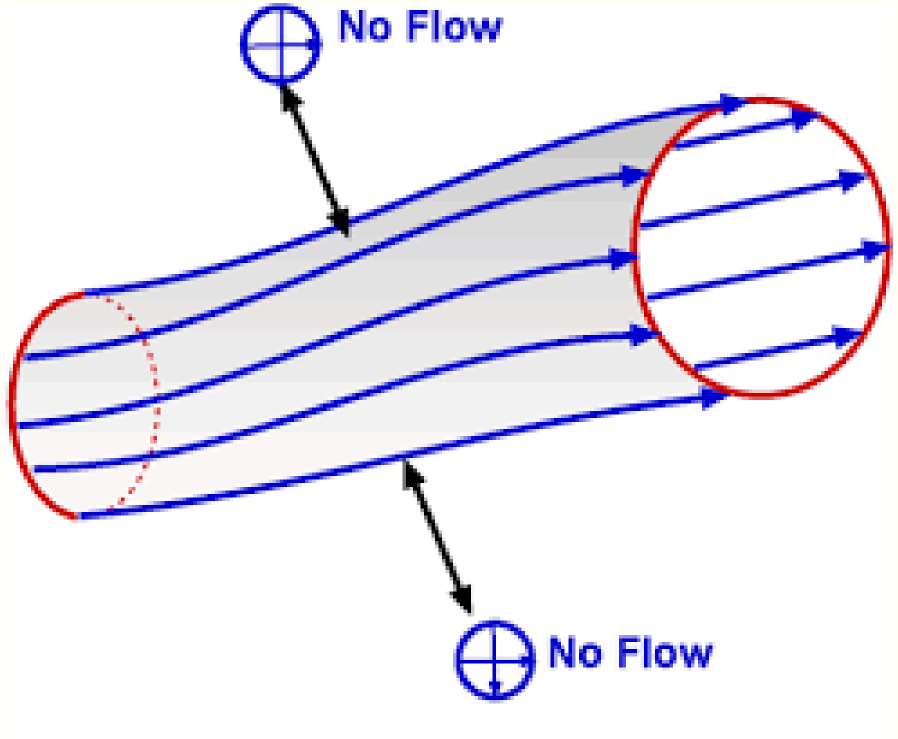
- A line which is everywhere tangent to the velocity vector at a given instant.
- It shows the mean direction of a number of particles at the same instant of time.



FLOW PATTERN

Streamtube:

- A bundle of streamline is called streamtube
- A stremtube is formed by a close collection of streamlines.
- Fluid can not flow in a direction perpendicular to the streamline
- Streamtube surface need not to be solid but may be fluid surface



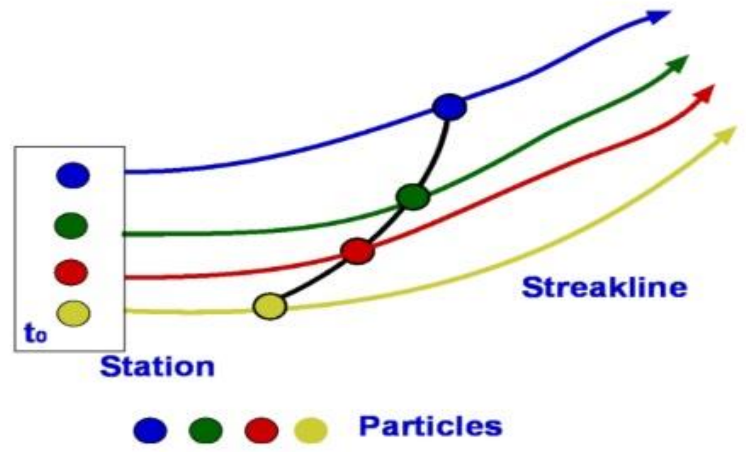
FLOW PATTERN

Pathline

- Is the trace made by a single particle over a period of time

Streakline:

Is the locus of a particle which earlier passed through a fixed point.



Streak line

Continuity Equation

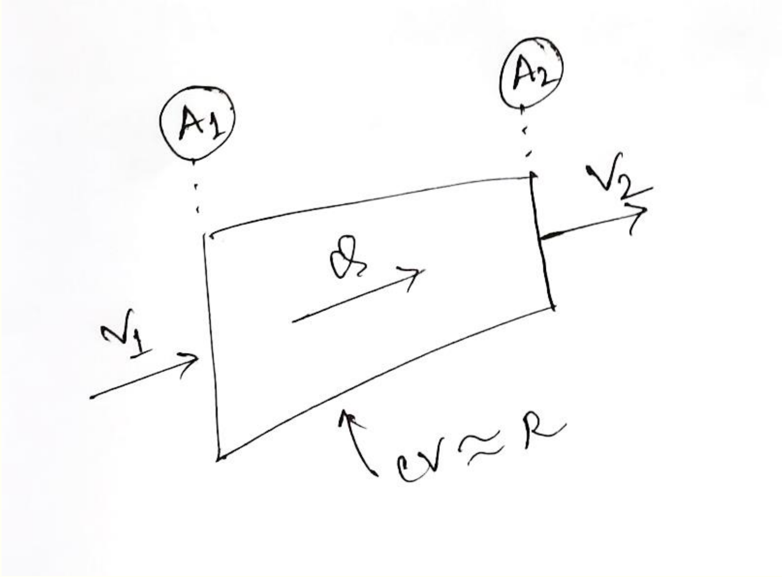
Expresses the conservation of Mass

M_t = mass of fluid contained in the control volume at time t

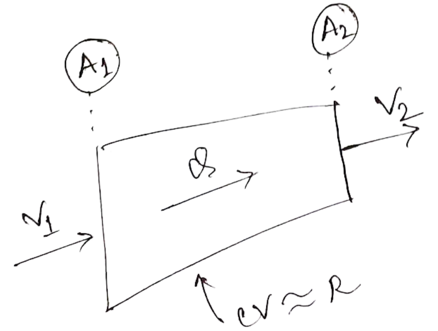
M_{t+dt} = mass of fluid contained in the control volume at time t+dt

$$M_{t+dt} = M_t + (\rho_1 v_1 \, dA_1) \, dt - (\rho_2 V_2 \, dA_2) \, dt$$

$$M_{t+dt} = M_t + \left(\frac{\partial \rho}{\partial t}\right) dt \, (R)$$



Continuity Equation



$$M_t + (\rho_1 v_1 dA_1)\,dt - (\rho_2 v_2 dA_2)\,dt = M_t + \left(\frac{\delta \rho}{\delta t}\right) dt\,(R)$$

$$\rho_1 v_1 dA_1 - \rho_2 v_2 dA_2 = \frac{\delta \rho}{\delta t} \cdot (R)$$

$$\rho_1 \int_{A_1} v_1 \, dA_1 - \rho_2 \int_{A_2} v_2 \, dA_2 = \int_R \frac{\delta \rho}{\delta t} \, dR.$$

Problems on Continuity Equation

A pipeline carries oil (relative density 0.86) at v = 2 m/s through 200mm cross-section. Find

- Mass flow rate
- Weight flow rate
- Volume flow rate
- Velocity in 60mm cross-section.

Problems on Continuity Equation

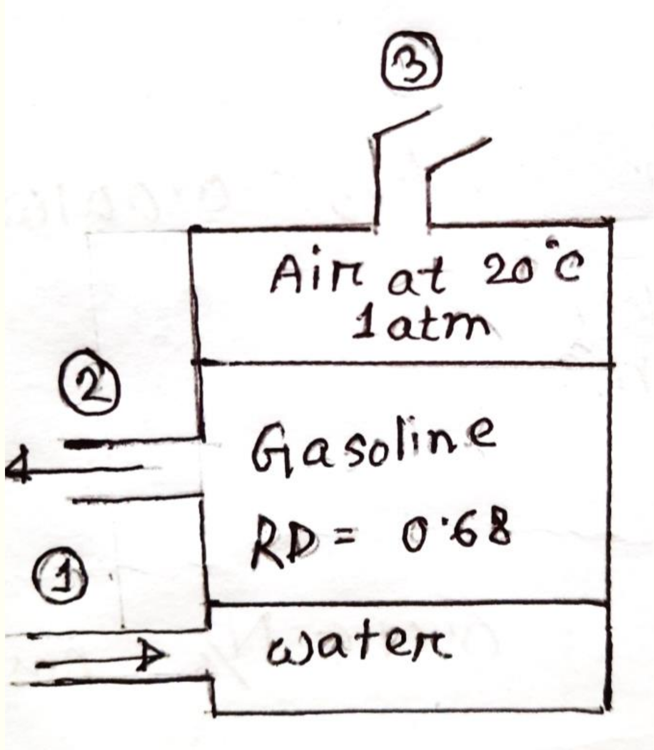
In steady viscous flow through a circular pipe the velocity profile is approximated by,

$$u = Um[1 - (\frac{r}{r_0})^2],$$

Compute the average velocity if the density is constant.

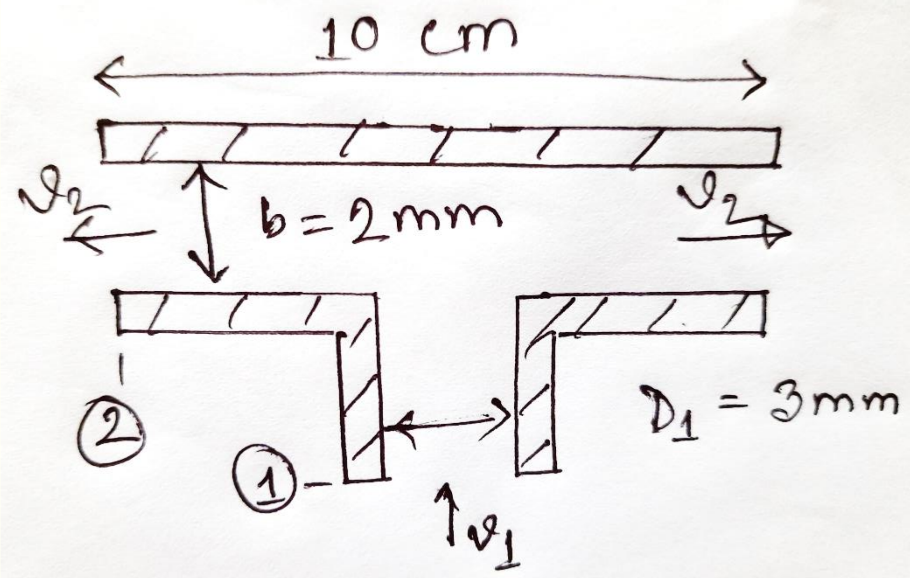
Problems on Continuity Equation

The tank in the figure admitting water at 90 N/s and ejecting gasoline (RD = 0.68) at 50 N/s. If all the fluids are incompressible, how much air is passing through the vent and in what direction? $\gamma_{air} = 0.0118\ KN/m3$



Problems on Continuity Equation

Oil enters at section 1 at 0.06N/s to lubricate the thrust bearing. The 10cm diameter bearing plates are 2mm apart. Assume steady flow compute the inlet velocity V_1 and outlet velocity V_2.



Velocity and acceleration in steady flow

$$u_{st} = u(x, y, z)$$

$$v_{st} = v(x, y, z)$$

$$w_{st} = w(x, y, z)$$

$$\mathbf{a}_{st} = u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}$$

$$\mathbf{a}_{st} = \frac{d}{dt}\mathbf{V}(x, y, z) = \frac{\partial \mathbf{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \mathbf{V}}{\partial y}\frac{dy}{dt} + \frac{\partial \mathbf{V}}{\partial z}\frac{dz}{dt}$$

$$|\mathbf{V}| = (u^2 + v^2 + w^2)^{1/2}$$

$$(a_x)_{st} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$(a_y)_{st} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$(a_z)_{st} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Velocity and acceleration in unsteady flow

$$a_x = \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) + \frac{\partial u}{\partial t}$$

$$a_y = \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) + \frac{\partial v}{\partial t}$$

$$a_z = \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) + \frac{\partial w}{\partial t}$$

Problems

A two dimensional flow field is given by u = 2y, v = x. Derive a general expression for the velocity and acceleration. Find the acceleration in the flow field at point A (x = 3.5, y = 1.2)

$$V = (u^2 + v^2)^{1/2} = (4y^2 + x^2)^{1/2}$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 2y(0) + x(2) = 2x$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 2y(1) + x(0) = 2y$$

$$a = (a_x^2 + a_y^2)^{1/2} = (4x^2 + 4y^2)^{1/2}$$

CHAPTER 4

ENERGY CONSIDERATION IN STEADY FLOW

First law of thermodynamics says –

Energy can neither be created nor destroyed

All forms of energy are equivalent

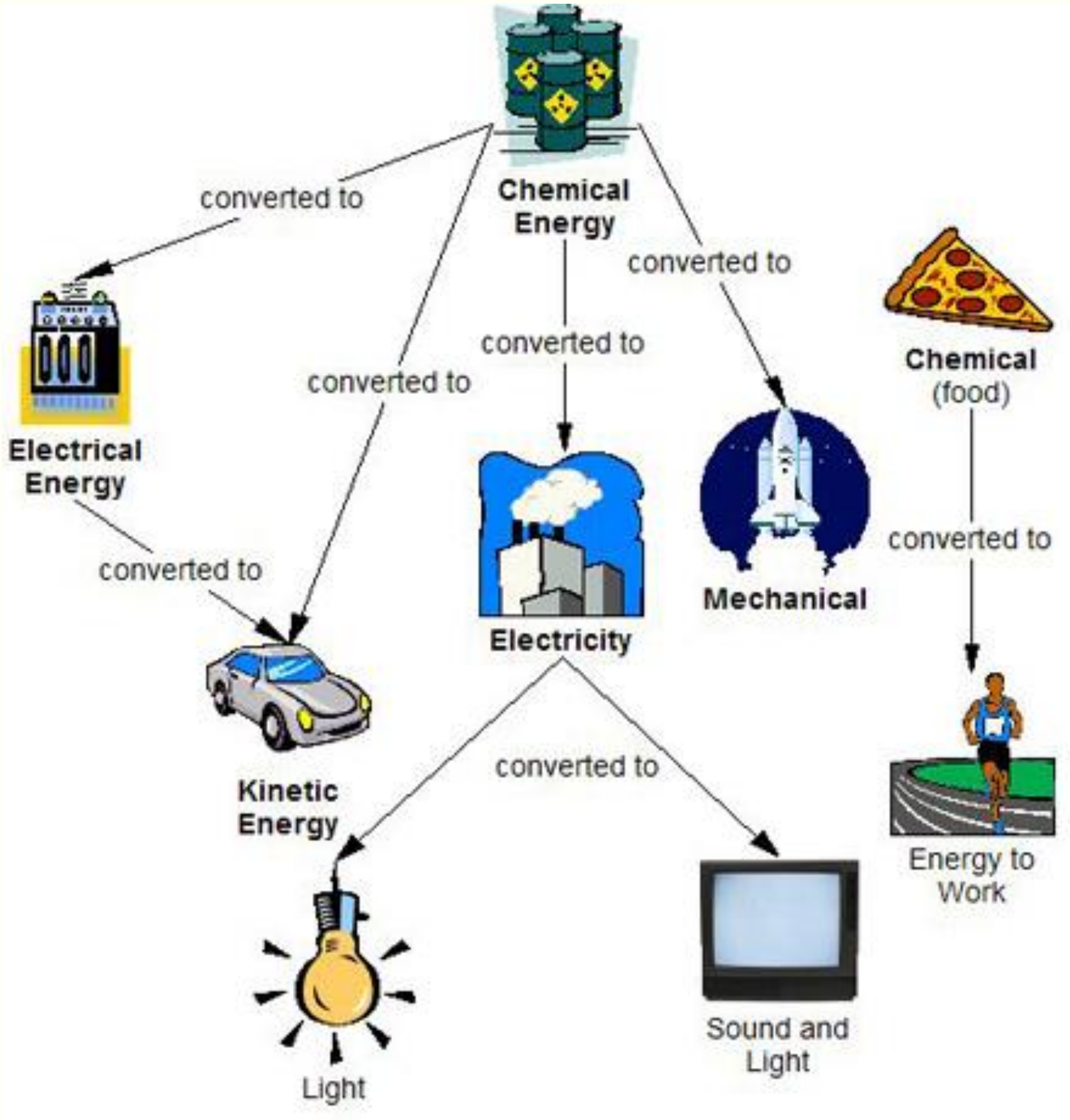


Figure collected from internet

Kinetic Energy of Flowing Fluid

A body of mass m flowing with a velocity V possesses a kinetic energy,

$$KE = \frac{1}{2}mv^2$$

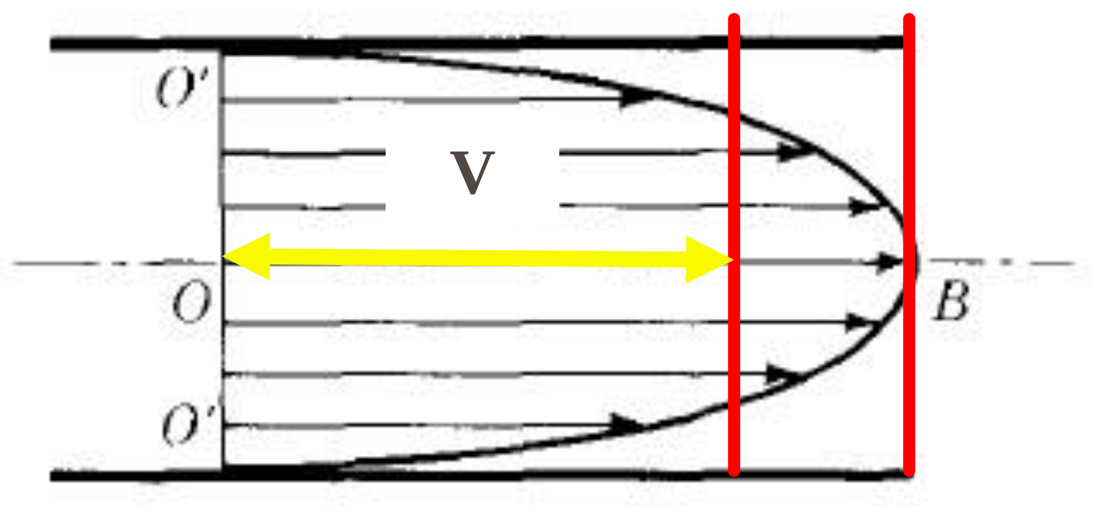
$$\frac{KE}{Weight} = \frac{\frac{1}{2}mv^2}{mg}$$

$$\frac{KE}{Weight} = \frac{v^2}{2g}$$

Kinetic Energy of Flowing Fluid

The velocities of the different real fluid particles are not the same, so it is necessary to integrate all velocity component along any direction of the stream to obtain the true value of the kinetic energy

$$\frac{KE}{Weight} = \alpha \frac{V^2}{2g}$$



(b) Real fluid.

Kinetic Energy of Flowing Fluid

To obtain the expression for α, let's consider velocity component u vary along the vertical direction.

The mass flow per unit of time through an elementary area dA is $\rho dQ = \rho udA$

Flow of kinetic energy per unit of time across area dA $= \frac{1}{2}\rho udAu^2 = \frac{1}{2}\rho u^3 dA$

Weight flow rate through dA = $\gamma Q = \gamma udA$

$$\frac{KE/time}{Weight/time} = \frac{\frac{1}{2g}\gamma u^3 dA}{\gamma udA} = \frac{\frac{1}{2g}u^3 dA}{udA} = \frac{\frac{1}{2g}\int u^3 dA}{\int udA}$$

$\alpha = 2$, for laminar flow in circular pipe

$\alpha = 1.01 - 1.15$, for turbulent flow

$$\alpha\frac{V^2}{2g} = \frac{\frac{1}{2g}\int u^3 dA}{\int udA} \qquad \alpha = \frac{\int u^3 dA}{V^2\int udA} = \frac{1}{AV^3}\int u^3\, dA$$

Average of cube is greater than the cube of the average, the value of α will always be greater than 1

Potential Energy of Flowing Fluid

Depends on elevation above any arbitrary datum.

A fluid particle of weight W situated a distance z above datum has potential energy of

$$PE = Wz$$

$$\frac{PE}{Weight} = \frac{Wz}{W} = z$$

Internal Energy of Flowing Fluid

It is a function of temperature.

Since we are concerned only with difference of energy, so $\Delta i = Cv\ \Delta T$

Where $Cv =$ specific heat at constant volume.

specific heat at constant volume, Cv: the amount of heat that is required to raise the temperature of unit mass of gas by 1 degree at **constant volume**.

General Equation for Steady Flow of Any Fluid

First Law of thermodynamics states:

For *steady flow* the external work done on any system plus the thermal energy transferred into or out of that system is equal to the change of energy of the system

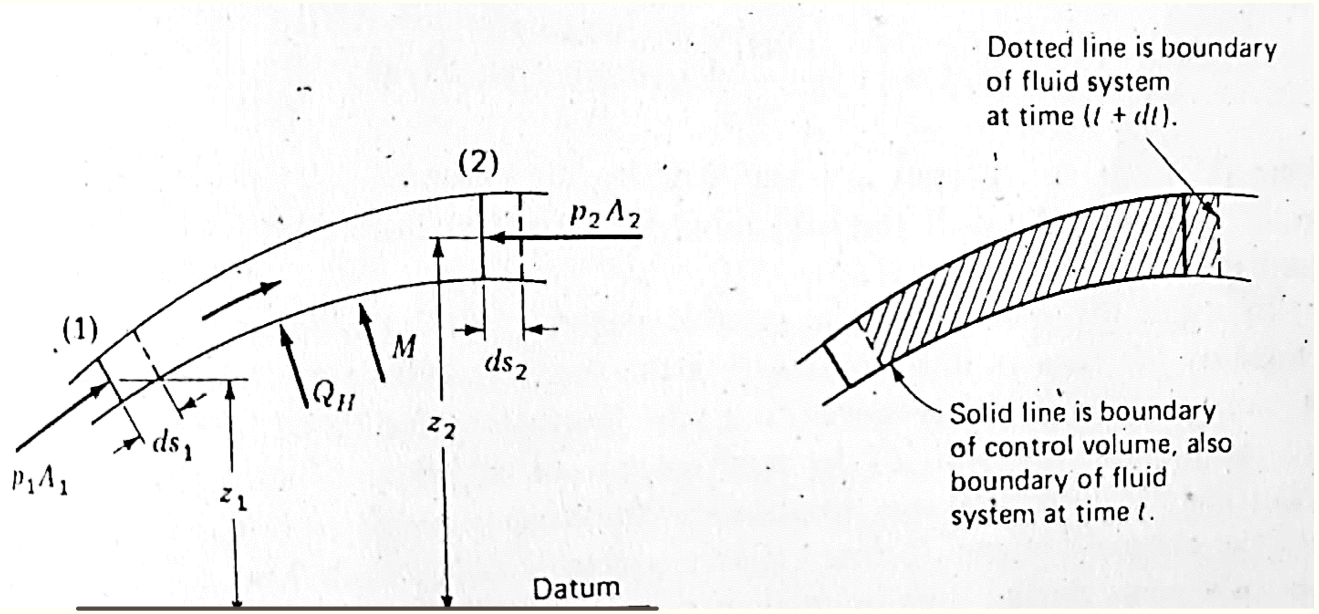
Work + Heat = Δ Energy

General Equation for Steady Flow of Any Fluid

Apply the first law of thermodynamics to the fluid system of this control volume

Control volume is fixed and does not move or change shape

For steady flow
$\gamma_1 A_1 ds_1 = \gamma_2 A_2 ds_2$



Flow work = $p_1 A_1 ds_1 - p_2 A_2 ds_2$

$$\text{Shaft work} = \frac{\text{weight}}{\text{time}} \times \frac{\text{energy}}{\text{weight}} \times \text{time}$$

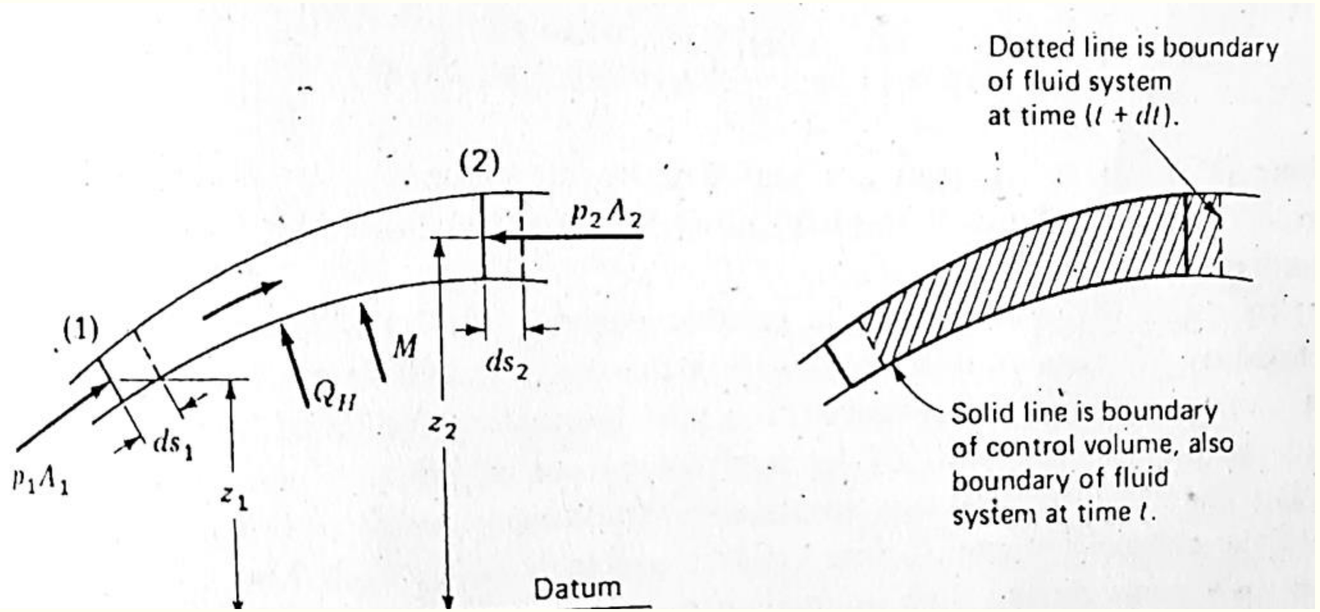
$$= \left(\gamma_1 A_1 \frac{ds_1}{dt}\right) h_M \, dt = (\gamma_1 A_1 \, ds_1) h_M$$

hm is the energy put into the flow by machine per unit weight of flowing fluid

General Equation for Steady Flow of Any Fluid

$$\text{Heat} = \left(\gamma_1 A_1 \frac{ds_1}{dt}\right) Q_H \, dt = (\gamma_1 A_1 \, ds_1) Q_H$$

$$\gamma_1 A_1 \, ds_1 (z_1 + \alpha V_1^2/2g + I_1),$$



$$\Delta \text{ energy} = \Delta E = \gamma_2 A_2 \, ds_2 \left(z_2 + \alpha_2 \frac{V_2^2}{2g} + I_2\right) - \gamma_1 A_1 \, ds_1 \left(z_1 + \alpha_1 \frac{V_1^2}{2g} + I_1\right)$$

General Equation for Steady Flow of Any Fluid

$$\frac{p_1}{\gamma_1} - \frac{p_2}{\gamma_2} + h_M + Q_H = \left(z_2 + \alpha_2 \frac{V_2^2}{2g} + I_2\right) - \left(z_1 + \alpha_1 \frac{V_1^2}{2g} + I_1\right)$$

$$\left(z_1 + \frac{p_1}{\gamma_1} + \alpha_1 \frac{V_1^2}{2g} + I_1\right) + h_M + Q_H = \left(z_2 + \frac{p_2}{\gamma_2} + \alpha_2 \frac{V_2^2}{2g} + I_2\right).$$

General Energy Equation

Bernoulli's Equation

$$\left(z_1 + \frac{p_1}{\gamma_1} + \alpha_1 \frac{V_1^2}{2g} + I_1\right) + h_M + Q_H = \left(z_2 + \frac{p_2}{\gamma_2} + \alpha_2 \frac{V_2^2}{2g} + I_2\right).$$

$$Z + \frac{P}{\gamma} + \frac{V^2}{2g} = constant$$

Assumptions:

- Non viscous fluid
- Incompressible fluid
- Uniform Velocity Distribution
- No mechanical energy added along the streamline

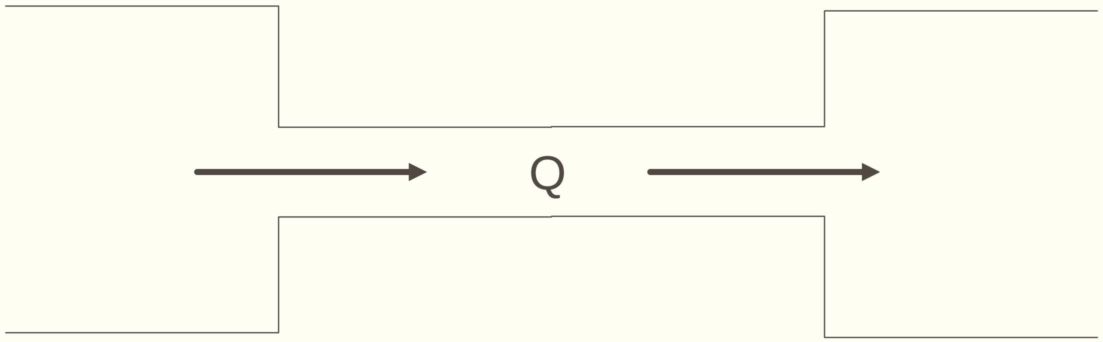
Head

Each term in the Bernoulli's equation has the dimension of length.

- Pressure Head $= \frac{P}{\gamma}$
- Velocity Head = $\frac{V^2}{2g}$
- Elevation Head = z

$$Total\ head,\ H = Z + \frac{P}{\gamma} + \frac{V^2}{2g}$$

Problems on Bernoulli's Equation



Problems on Bernoulli's Equation

Consider Flow through a horizontal conical diffuser with following details:

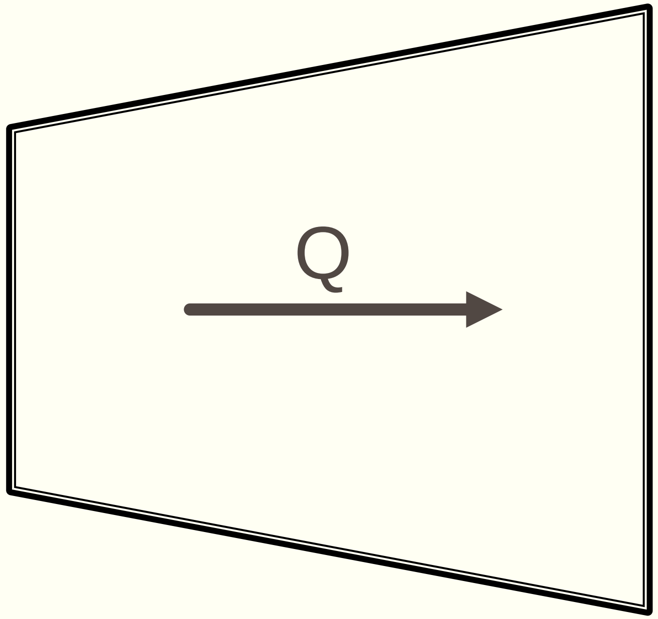
$Q = 5\ m^3/s$

$D1 = 1.2m$

$D2 = 1.6m$

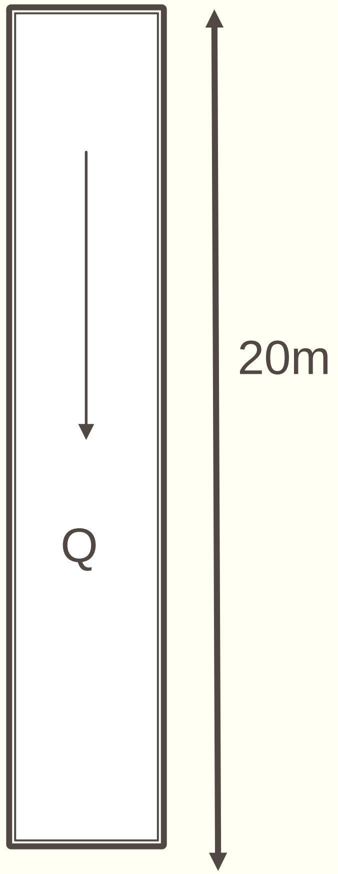
$P1 = 7.5\ kN/m^2$

$P2 = ?$



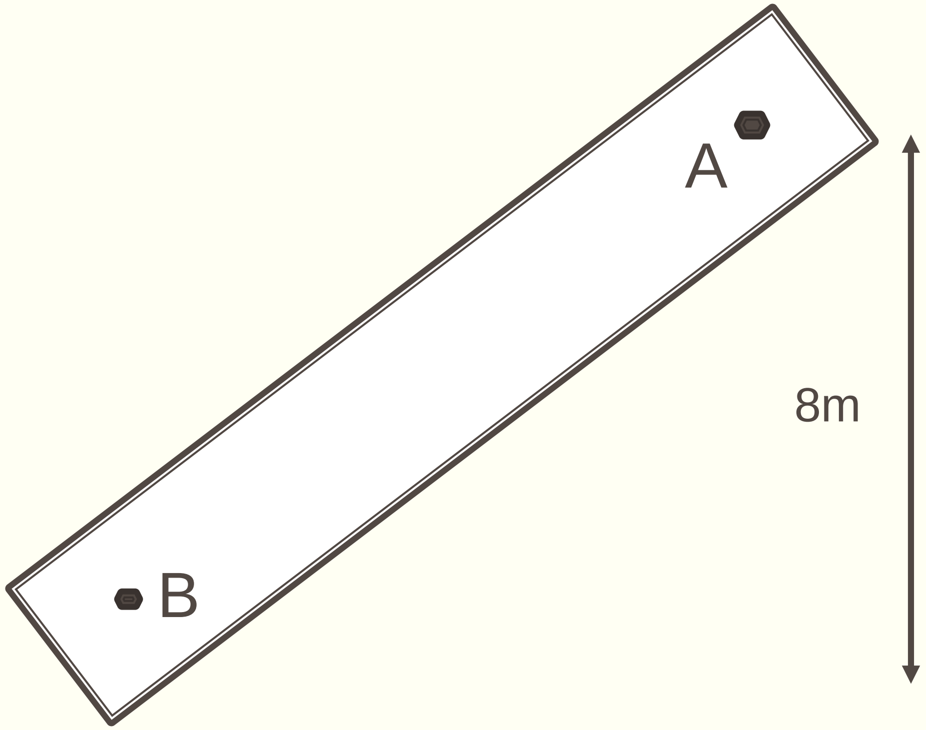
Problems on Energy Equation

Upstream pressure head and velocity for the flow to the downward direction are 5m and 5m/s respectively though the circular pipe of 2m diameter. Find downstream pressure head if the headloss encountered thorough the flow path is 1.25m.



Problems on Energy Equation

A liquid of Sp Gr 0.85 is flowing through a pipe of uniform diameter. Pressure head at point A and B is 150kN/m and 250kN/m. Determine the flow direciton and the headloss through the pipe.

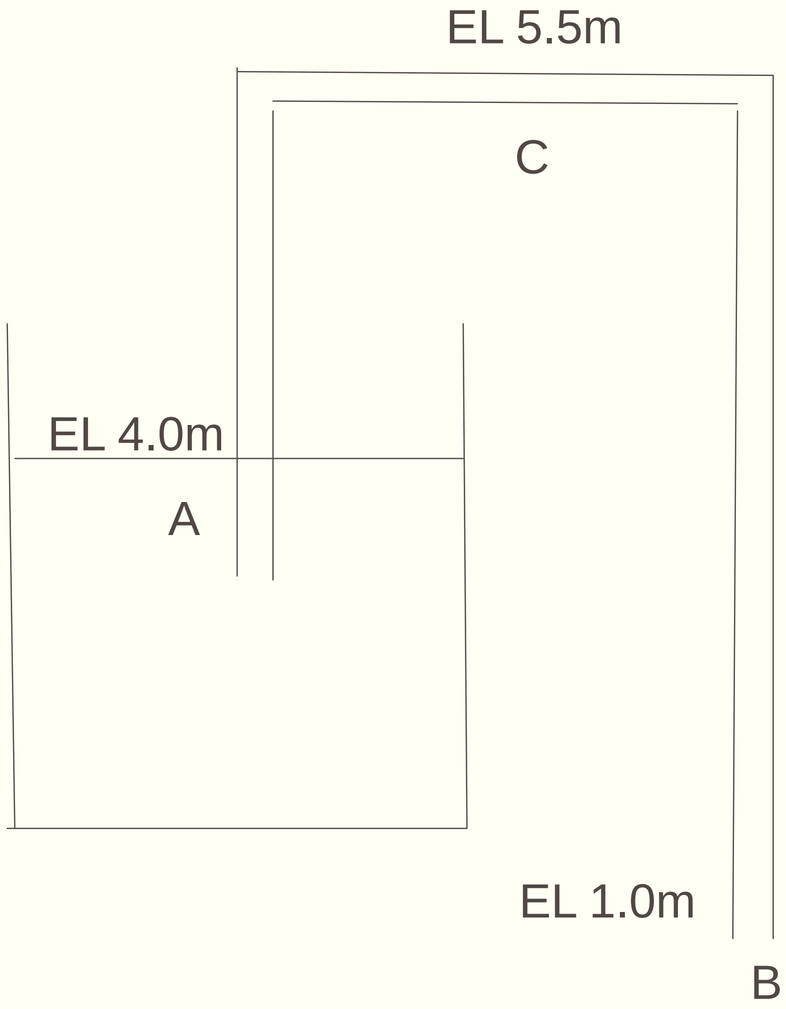


Problems on Energy Equation

Consider the flow through a syphon consisting a pipe of 15 cm diameter.

- ➢HL (A-C) = 0.5m, HL (C-B) = 1.2m
- ➢RD of Fluid = 0.8

Find the discharge, Q and Pressure at Point C



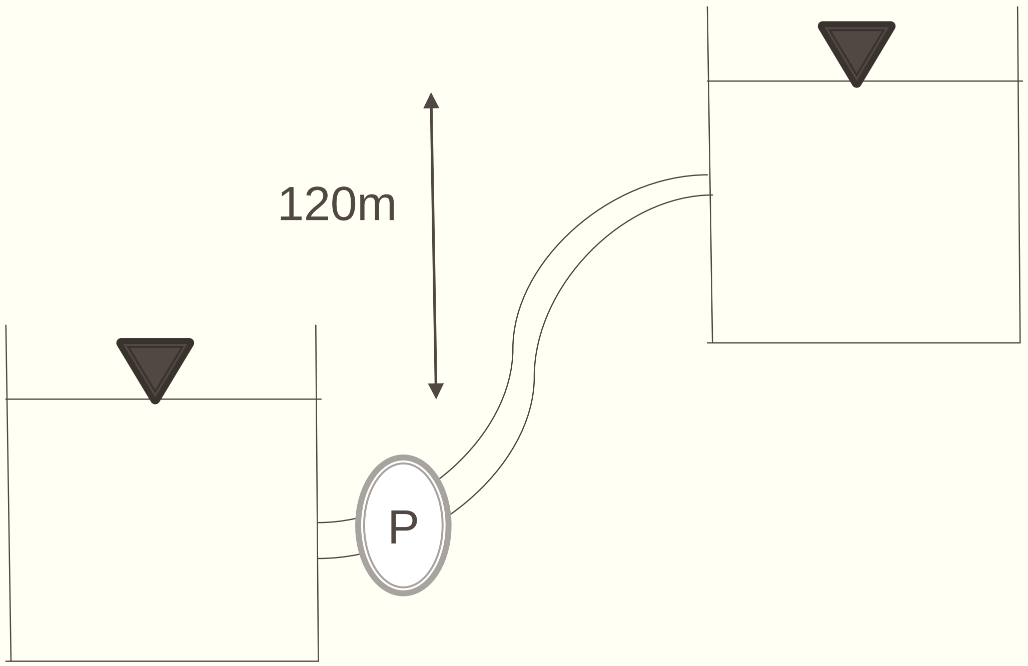
Power Consideration in Fluid Flow

$$Power = \frac{Energy}{time} = \frac{Energy}{Weight} \times \frac{Weight}{time} = H\gamma Q$$

1HP = 746 W

Problem

A pump lifts water at the rate of 6 m3/s to a height of 120m. The friction loss in the pipe is 10m. What is the horsepower required if the pump efficiency is 90%



Problem

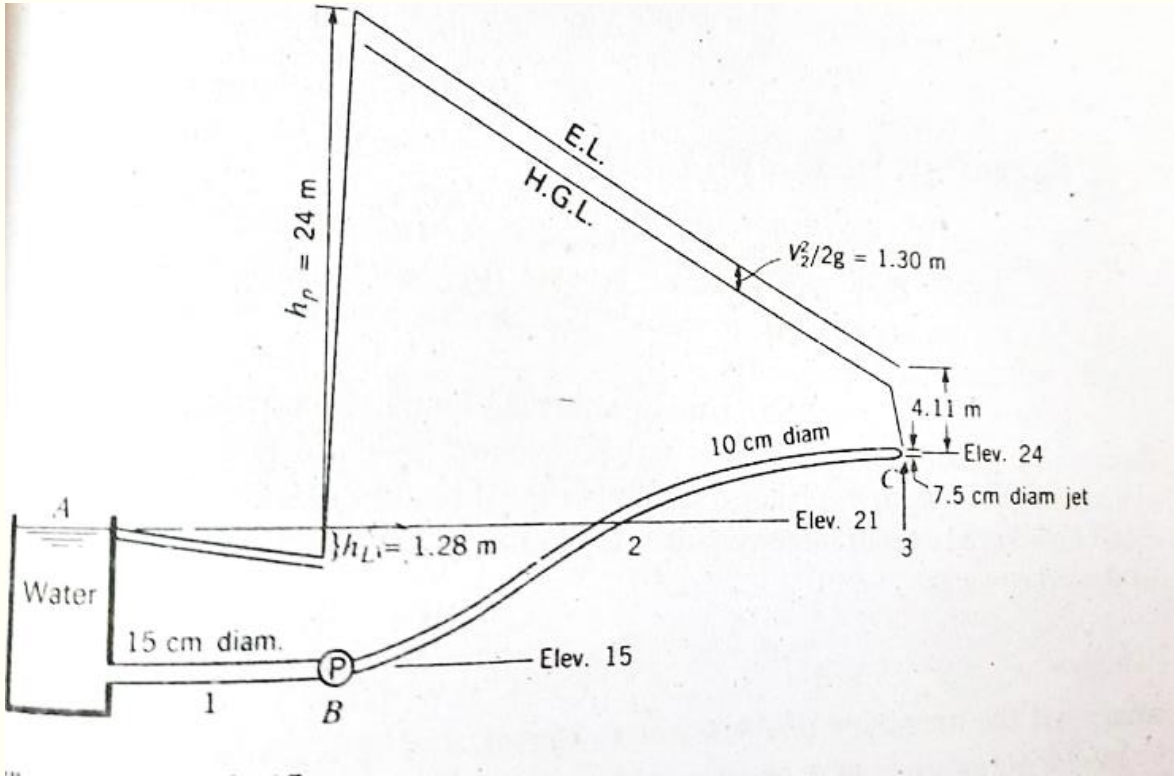
A turbine is set 40m below the water level of a reservoir and is fed by a 60cm diameter pipe. A short pipe of 45cm diameter discharges the water from the turbine to the atmosphere. Find:

a. Neglecting friction, estimate the power extracted by the turbine when the discharge is 0.8 m^3/sec.

b. If a total friction loss of 10m is assumed on the turbine efficiency is 85%, estimate the power output.

Problem

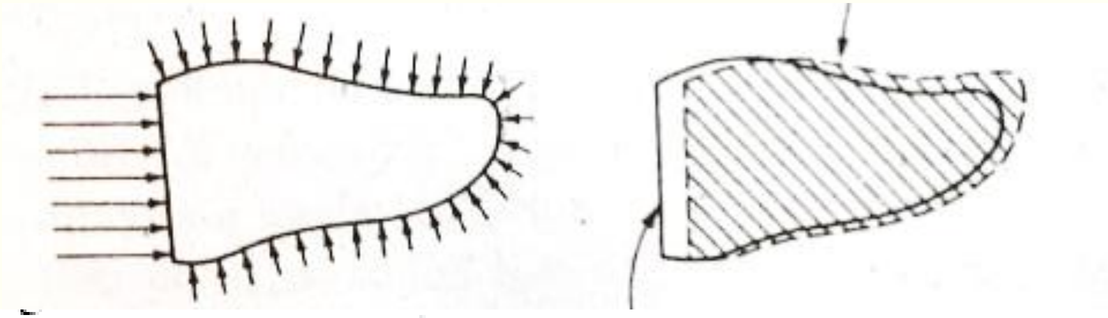
A pipeline with a pump leads to a nozzle. Find the flow rate when the pump develops a head of 24m. Assume that the head loss in the 15cm diameter pipe may be expressed by $h_L = 5\frac{{V_1}^2}{2g}$, while the head loss in the 10cm diameter pipe is $h_L = 12\frac{{V_2}^2}{2g}$. Sketch the energy line and hydraulic grade line, and find the pressure had at the suction side of the pump.



CHAPTER 6

Momentum and Forces In Fluid Flow

Development of Impulse-Momentum principle



$(mV)_t$ = momentum at time t of the fluid system (coincident with the control volume at time t)

$(mV)_{t+\Delta t}$ = momentum at time $(t + \Delta t)$ of the fluid system (coincident with the shaded area of Fig. 6.1 at time $t + \Delta t$)

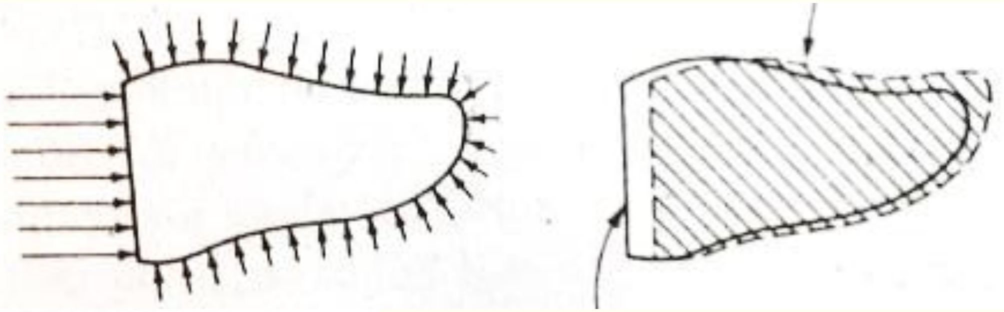
$(m'V')_t$ = momentum of the fluid mass contained within the control volume at time t

$(m'V')_{t+\Delta t}$ = momentum of the fluid mass contained within the control volume at time $(t + \Delta t)$

$\Delta(mV)_{out}$ = momentum of the fluid mass that leaves the control volume during time interval Δt

$\Delta(mV)_{in}$ = momentum of the fluid mass that enters the control volume during time interval Δt

Development of Impulse-Momentum principle



$$(m\mathbf{V})_{t+\Delta t} = (m'\mathbf{V}')_{t+\Delta t} + \Delta(m\mathbf{V})_{\text{out}} - \Delta(m\mathbf{V})_{\text{in}}$$

The change of momentum of the fluid system is

$$\Delta(m\mathbf{V}) = (m\mathbf{V})_{t+\Delta t} - (m\mathbf{V})_t$$

Substituting the two preceding expressions into Eq. (6.2), we get

$$\Delta(m\mathbf{V}) = (m'\mathbf{V}')_{t+\Delta t} - (m'\mathbf{V}')_t + \Delta(m\mathbf{V})_{\text{out}} - \Delta(m\mathbf{V})_{\text{in}}$$

Applying Eq. (6.1), dividing through by Δt, rearranging, and noting that the limit of $\Delta(m\mathbf{V})/\Delta t = d(m\mathbf{V})/dt$ as $\Delta t \to 0$, we get

$$\sum \mathbf{F} = \lim_{t \to 0} \frac{\Delta(m\mathbf{V})}{\Delta t} = \frac{d(m\mathbf{V})}{dt}$$

$$= \frac{d(m\mathbf{V})_{\text{out}} - d(m\mathbf{V})_{\text{in}}}{dt} + \frac{(m'\mathbf{V}')_{t+\Delta t} - (m'\mathbf{V}')_t}{dt} \qquad (6.3)$$

Development of Impulse-Momentum principle

$$\sum \mathbf{F} = \frac{d(m\mathbf{V})_{out} - d(m\mathbf{V})_{in}}{dt} = \frac{d(m\mathbf{V})_{out}}{dt} - \frac{d(m\mathbf{V})_{in}}{dt}$$

Development of Impulse-Momentum principle

$$\sum F_x = \rho_2 Q_2 V_{2_x} - \rho_1 Q_1 V_{1_x} = \rho Q(\Delta V_x)$$

$$\sum F_y = \rho_2 Q_2 V_{2_y} - \rho_1 Q_1 V_{1_y} = \rho Q(\Delta V_y)$$

$$\sum F_z = \rho_2 Q_2 V_{2_z} - \rho_1 Q_1 V_{1_z} = \rho Q(\Delta V_z)$$

Problems

Mathematical questions will be practiced from text books